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CHAMBER CARBON DIOXIDE AND VENTILATION



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GLOSSARY

Δ_{CO_2}	chamber CO ₂ partial pressure difference between values at the beginning and end of a no-ventilation – ventilation cycle
$\Delta_{CO_2,i}$	chamber CO ₂ partial pressure difference between value at the beginning of a series of no-ventilation – ventilation cycles and the value at the end of cycle i
$\Delta P_{CO_2}(t_v)_i$	change in chamber CO ₂ partial pressure during ventilation in no-ventilation – ventilation cycle i
F	CO ₂ scrubber efficiency factor
i	cycle number in a series of no-ventilation – ventilation cycles
k	ventilation time constant with chamber (or scrubber, depending on context) volumetric ventilation rate at standard temperature and pressure (STP)
k_A	ventilation time constant with chamber (or scrubber, depending on context) volumetric ventilation rate at chamber temperature and pressure
n_{CO_2}	chamber CO ₂ content, moles
n_g	number of moles of gas species g
n_T	chamber gas content, moles
\dot{n}_{CO_2}	molar rate of change of chamber CO ₂ content
$\dot{n}_{CO_2,in}$	molar rate of CO ₂ input to chamber via input ventilation
$\dot{n}_{CO_2,met}$	molar rate of CO ₂ input to chamber via metabolism
$\dot{n}_{CO_2,scrub}$	molar rate of CO ₂ removal from chamber via scrubbing
$\dot{n}_{CO_2,out}$	molar rate of CO ₂ removal from chamber via exhaust ventilation
$\dot{n}_{g,met}$	molar production or consumption rate of gas species g by metabolism

$\dot{n}_{g,scrub}$	molar consumption rate of gas species g by scrubbing
$\dot{n}_{O_2,met}$	molar rate of metabolic O_2 consumption
$\dot{n}_{T,in}$	total molar rate of chamber input ventilation
$\dot{n}_{T,out}$	total molar rate of chamber exhaust ventilation
P_{ch}	chamber pressure
P_{CO_2}	chamber CO_2 partial pressure
$P_{CO_2}^o$	initial chamber CO_2 partial pressure
$P_{CO_2,vin}$	CO_2 partial pressure in input ventilation gas
$P_{CO_2}(t_{nv})_i$	chamber CO_2 partial pressure at end of no-ventilation period in chamber no-ventilation – ventilation cycle i
$P_{CO_2}(t_v)_i$	chamber CO_2 partial pressure at end of ventilation period in chamber no-ventilation – ventilation cycle i
$P_{CO_2}(\infty)$	steady-state CO_2 partial pressure in a continuously ventilated chamber
P_{ex}	pressure of exhaust ventilation gas
P_{in}	pressure of input ventilation gas
P_s	standard pressure
Q	$(1-q)$; remaining fraction of total response
q	fraction of total response
R	gas constant
RQ	respiratory quotient; ratio of CO_2 produced to O_2 consumed
T_{ch}	chamber temperature
T_s	standard temperature

t	time
t_{∞}	infinite time
$t_{1/2}$	half-time
t_{nv}	chamber no-ventilation time
$(t_{nv})_i$	no-ventilation time in chamber no-ventilation – ventilation cycle i
t_q	time for q response
t_v	chamber ventilation time
$t_{v,1/2}$	chamber ventilation half-time
$(t_v)_i$	ventilation time in chamber no-ventilation – ventilation cycle i
V_{ch}	chamber volume
$\dot{V}_{CO_2,in}$	rate of CO ₂ input via ventilation at chamber temperature and pressure
$\dot{V}_{CO_2,in,STP}$	rate of CO ₂ input via ventilation at standard temperature and pressure
$\dot{V}_{CO_2,scrub,STP}$	rate of CO ₂ uptake via scrubbing at standard temperature and pressure
\dot{V}_{vin}	overall input ventilation rate at chamber temperature and pressure
$\dot{V}_{vin,STP}$	overall input ventilation rate at standard temperature and pressure
$\dot{V}_{vin,scr}$	overall CO ₂ scrubber input ventilation rate at chamber temperature and pressure
$\dot{V}_{CO_2,met}$	metabolic CO ₂ production rate at chamber temperature and pressure
$\dot{V}_{CO_2,met,STP}$	metabolic CO ₂ production rate at standard temperature and pressure
$\dot{V}_{CO_2,out}$	CO ₂ exhaust rate via ventilation at chamber temperature and pressure
$\dot{V}_{CO_2,out,STP}$	CO ₂ exhaust rate via ventilation at standard temperature and pressure

\dot{V}_{vex}	overall exhaust rate at chamber temperature and pressure
$\dot{V}_{vex,STP}$	overall exhaust rate at standard temperature and pressure
$\dot{V}_{O_2,met}$	metabolic O ₂ consumption rate at chamber temperature and pressure
$\dot{V}_{O_2,met,STP}$	metabolic O ₂ consumption rate at standard temperature and pressure
X_{CO_2}	CO ₂ mole fraction

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1. INTRODUCTION

Maintenance of atmospheric carbon dioxide (CO_2) concentrations within physiologically acceptable limits is critical to the safe operation of hyperbaric chambers and manned diving systems. Quantitative understanding of CO_2 accumulation in such systems is required to specify minimum ventilation rates and CO_2 scrubbing capacities, but existing theoretical treatments that we could find in the literature are few and incomplete. For example, Nuckols *et al.*¹ present an equation giving the time course of CO_2 partial pressure (P_{CO_2}) change in a continuously ventilated chamber, but the equation is valid only if the net respiratory quotient (RQ) of all chamber occupants is unity and the initial P_{CO_2} is zero. The latter constraint precludes consideration of the maximum chamber P_{CO_2} attained during realistic intermittent ventilation scenarios. In this report we present a more general solution to the problem with expressions for computing the maximum chamber P_{CO_2} attained during intermittent ventilation, and for estimating the time course of CO_2 accumulation in chambers with an internal scrubber that continuously removes CO_2 .

2. THEORY

2.1. Basic Equations

If we assume that all gas in the problem is ideal, the total chamber gas content in moles is

$$n_T = \frac{P_{ch} V_{ch}}{RT_{ch}}, \quad (1)$$

where P_{ch} is chamber pressure, V_{ch} is chamber volume, T_{ch} is chamber temperature, and R is the gas constant. The chamber CO_2 content is

$$n_{\text{CO}_2} = X_{\text{CO}_2} n_T = \frac{X_{\text{CO}_2} P_{ch} V_{ch}}{RT_{ch}}, \quad (2)$$

where X_{CO_2} is the CO_2 mole fraction in the chamber. The CO_2 partial volume V_{CO_2} and partial pressure P_{CO_2} in the chamber are then obtained from Dalton's Law —

$$V_{\text{CO}_2} = X_{\text{CO}_2} V_{ch} \quad (3.a)$$

and

$$P_{\text{CO}_2} = X_{\text{CO}_2} P_{ch} \quad (3.b)$$

— and are substituted into Eq. (2) to yield, respectively:

$$n_{CO_2} = \frac{P_{ch} V_{CO_2}}{RT_{ch}} \quad (4.a)$$

and

$$n_{CO_2} = \frac{V_{ch} P_{CO_2}}{RT_{ch}}. \quad (4.b)$$

We express V_{CO_2} at standard temperature and pressure (STP), T_S and P_S , respectively:

$$V_{CO_2,STP} = \frac{P_{ch} T_S}{P_S T_{ch}} V_{CO_2}. \quad (5)$$

We then substitute Eq. (5) into Eq. (4.a) to yield

$$n_{CO_2} = \frac{P_S V_{CO_2,STP}}{RT_S}. \quad (4.c)$$

The rate of change of chamber CO_2 content is obtained by differentiating Eq. (4.b) at constant chamber temperature and volume to obtain

$$\left(\frac{dn_{CO_2}}{dt} \right)_{T_{ch}} = \frac{V_{ch}}{RT_{ch}} \left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} \equiv \frac{V_{ch} \dot{P}_{CO_2}}{RT_{ch}}, \quad (6.a)$$

or by differentiating Eq. (4.c) to obtain

$$\left(\frac{dn_{CO_2}}{dt} \right) = \frac{P_S}{RT_S} \left(\frac{dV_{CO_2,STP}}{dt} \right) \equiv \frac{P_S \dot{V}_{CO_2,STP}}{RT_S}. \quad (6.b)$$

Finally, mass balance requires that the change in chamber CO_2 content be equal to the sum of changes from the chamber input and exhaust ventilation, the amount of metabolic CO_2 produced by the occupants, and the amount of CO_2 scrubbed by chemical means in the chamber:

$$\dot{n}_{CO_2} = \dot{n}_{CO_2,in} + \dot{n}_{CO_2,met} - \dot{n}_{CO_2,scrub} - \dot{n}_{CO_2,out}, \quad (7)$$

where $\dot{n}_{CO_2} \equiv \left(\frac{dn_{CO_2}}{dt} \right)$. Using Eq. (6.b), Eq. (7) becomes

$$\dot{V}_{CO_2,STP} = \dot{V}_{CO_2,in,STP} + \dot{V}_{CO_2,met,STP} - \dot{V}_{CO_2,scrub,STP} - \dot{V}_{CO_2,out,STP}, \quad (8)$$

where $\dot{V}_{CO_2,met,STP}$ is the standard rate of metabolic CO_2 production by the chamber occupants. All quantities on the right of Eq. (8), except $\dot{V}_{CO_2,scrub,STP}$ when $\dot{V}_{CO_2,scrub,STP} > 0$, are assumed to be

constant under each set of conditions considered in this analysis. The assumption of constant $\dot{V}_{CO_2,met,STP}$ requires a sufficiently high chamber P_{O_2} to sustain the assumed $\dot{V}_{CO_2,met,STP}$.

2.2. CO₂ Accumulation in a Nonventilated Constant Temperature Chamber

With no CO₂ flow into the chamber, no gas exhausted from the chamber, and no CO₂ scrubbing, the standard partial CO₂ volume in the chamber changes at a rate equal to $\dot{V}_{CO_2,met,STP}$:

$$\dot{V}_{CO_2,STP} = \dot{V}_{CO_2,met,STP} \quad (9)$$

Equation (9) is substituted into Eq. (6.b) to obtain

$$\left(\frac{dn_{CO_2}}{dt} \right) = \frac{P_S \dot{V}_{CO_2,met,STP}}{RT_S} \quad (10)$$

Equations (10) and (6.a) are then combined to obtain

$$\frac{V_{ch}}{RT_{ch}} \left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{P_S \dot{V}_{CO_2,met,STP}}{RT_S} \quad (11)$$

which is rearranged to yield

$$\left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} \quad (12)$$

The P_{CO_2} at time t in a no-ventilation period between $t = 0$ and $t = t_{nv}$ is given by integrating Eq. (12):

$$P_{CO_2}(t) = \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} \cdot t + P_{CO_2}^o \quad (13)$$

where $P_{CO_2}(t)$ is the P_{CO_2} at $0 \leq t \leq t_{nv}$, and $P_{CO_2}^o$ is the initial P_{CO_2} at $t = 0$.

2.3. CO₂ Accumulation/Washout in a Continuously Ventilated Constant Temperature Chamber

The rate of change in CO₂ content of a continuously ventilated constant temperature chamber with no scrubbing is obtained in terms of the CO₂ partial pressure from Eqs. (6.a) and (6.b), and the mass balance equation, Eq. (8), with $\dot{V}_{CO_2,scrub,STP} = 0$:

$$\left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{T_{ch} P_S}{T_S V_{ch}} (\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} - \dot{V}_{CO_2,out,STP}). \quad (14)$$

Under the assumption that the chamber is well stirred, we then have that

$$\dot{V}_{CO_2,out,STP} = X_{CO_2} \dot{V}_{vex,STP} = \frac{P_{CO_2}}{P_{ch}} \dot{V}_{vex,STP}, \quad (15)$$

where $\dot{V}_{vex,STP}$ is the ventilation exhaust rate at STP, and Eq. (3.b) has been used to express the mole fraction of chamber CO_2 in terms of the chamber P_{CO_2} and total pressure. Substitution of Eq. (15) into Eq. (14) yields

$$\left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} = \frac{T_{ch} P_S}{T_S V_{ch}} \left(\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} - \frac{P_{CO_2}}{P_A} \dot{V}_{vex,STP} \right), \quad (16)$$

which is rearranged to obtain

$$\left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} + \left(\frac{T_{ch} P_S \dot{V}_{vex,STP}}{T_S P_{ch} V_{ch}} \right) P_{CO_2} = \frac{T_{ch} P_S}{T_S V_{ch}} (\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}). \quad (17)$$

Note that Eq. (17) reduces to Eq. (12) when $\dot{V}_{vex,STP} = \dot{V}_{CO_2,in,STP} = 0$; the conditions assumed to derive Eq. (12) in Section 2.2. However, with constant $P_{ch} (> 0)$, $\dot{V}_{vex,STP} (> 0)$, and $\dot{V}_{CO_2,in,STP} (\geq 0)$, Eq. (17) is a linear first-order differential equation of form

$$y' + ky = c \quad (18)$$

with solution (see Section 3.1)

$$y = \frac{c(1 - e^{-kt})}{k} + C \cdot e^{-kt}, \quad (19)$$

where

$$y = P_{CO_2}(t), \quad (20.a)$$

$$k = \left(\frac{T_{ch} P_S \dot{V}_{vex,STP}}{T_S P_{ch} V_{ch}} \right), \quad (20.b)$$

$$c = \frac{T_{ch} P_S}{T_S V_{ch}} (\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}), \quad (20.c)$$

and C is a constant determined from initial conditions for the problem. Cancellation of terms after appropriate substitutions into Eq. (19) yields:

$$P_{CO_2}(t) = \frac{P_{ch} [\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}]}{\dot{V}_{vex,STP}} [1 - e^{-kt}] + C e^{-kt}. \quad (21)$$

Solution of Eq. (21) at $t = 0$ shows that $C = P_{CO_2}^o$, the partial pressure of CO_2 in the chamber at time 0, so that our final result is

$$P_{CO_2}(t) = \frac{P_{ch} [\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}]}{\dot{V}_{vex,STP}} [1 - e^{-kt}] + P_{CO_2}^o e^{-kt}, \quad \text{with } k = \frac{T_{ch} P_S \dot{V}_{vex,STP}}{T_S P_{ch} V_{ch}}; \quad (22.a)$$

and

$$P_{CO_2}(t) = \frac{P_S T_{ch} [\dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP}]}{T_S \dot{V}_{vex}} [1 - e^{-k_A t}] + P_{CO_2}^o e^{-k_A t}, \quad \text{with } k_A = \frac{\dot{V}_{vex}}{V_{ch}}; \quad (22.b)$$

where Eq. (22.b) applies when the ventilation rate is expressed as the volume flux at actual chamber temperature and pressure, T_{ch} and P_{ch} .

Note that V_{ch} is a constant with units of volume but without reference to any temperature or pressure, while the dependence of $P_{CO_2}(t)$ on P_{ch} evident in Eq. (22.a) is obscured in the units ratio, $\frac{V_{STP}}{V} = \frac{P_{ch} T_S}{P_S T_{ch}}$, in Eq. (22.b). In addition, maintenance of the constant P_{ch} required by either Eq. (22.a) or Eq. (22.b) requires a minimum constant gas bleed of $\dot{V}_{vin} = \dot{V}_{O_2,met}(1 - RQ)$ into the chamber, where RQ is the respiratory quotient (see Section 3.3). This gas bleed is insufficient to sustain any specified $\dot{V}_{CO_2,met}$ indefinitely. The minimum input ventilation of 100% O_2 required for this purpose, $\dot{V}_{vin,STP} = \dot{V}_{O_2,met,STP} = \dot{V}_{CO_2,met,STP} / RQ$, then requires $\dot{V}_{vex,STP} > 0$ to keep P_{ch} constant (see Section 3.3). The requirement for $\dot{V}_{vex,STP} > 0$ also becomes clear when we next consider the steady state P_{CO_2} approached at the other boundary condition on Eqs. (22.a) and (22.b): namely, that at $t = \infty$. Finally, if $RQ = 1$, maintenance of constant chamber pressure requires $\dot{V}_{vex,STP} = \dot{V}_{vin,STP}$ (see Section 3.3), from which it follows that $\dot{V}_{vex} = \dot{V}_{vin}$. If we also set $P_{CO_2}^o = 0$ and express the metabolic CO_2 production rate at chamber temperature and pressure, Eq. (22.b) becomes identical to that given by Nuckols, *et al.*:¹

$$P_{CO_2}(t) = \left[\frac{P_{ch} \dot{V}_{CO_2,met}}{\dot{V}_{vin}} + P_{CO_2,vin} \right] [1 - e^{-k_A t}], \quad \text{with } RQ = 1 \text{ and } P_{CO_2}^o = 0; \quad (23)$$

where $P_{CO_2,vin}$ is the CO_2 partial pressure in the input ventilation gas.

Steady-State P_{CO_2}

As $t \rightarrow \infty$, the P_{CO_2} in a continuously ventilated chamber approaches a steady-state value given by the solution of Eq. (22.a) or (22.b) at $t = \infty$:

$$P_{CO_2}(\infty) = \frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} \quad (24.a)$$

$$= \frac{P_S T_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{T_S \dot{V}_{vex}} \quad (24.b)$$

Because the chamber P_{CO_2} can never exceed the prevailing chamber pressure, the ratio $\{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \} / \dot{V}_{vex,STP}$ can never exceed unity, and $\dot{V}_{vex,STP} \geq \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}$. At the smallest value of $\dot{V}_{vex,STP}$ for which Eq. (24.a) remains applicable, or the smallest value of \dot{V}_{vex} for which Eq. (24.b) remains applicable, the steady-state chamber P_{CO_2} is simply the chamber pressure, P_{ch} . The latter result is a consequence of the well-stirred assumption, which applies to the metabolic processes in the chamber as well as to the chamber gases. At minimum \dot{V}_{vex} , the accompanying minimum O_2 influx, $\dot{V}_{vin}(100\% O_2) = \dot{V}_{O_2,met}$ (see Section 3.3) is instantaneously mixed and metabolized to CO_2 so that no O_2 remains.

Ventilation Half-Time

The ventilation half-time $t_{v,1/2}$ for approach to the steady state given by Eq. (24.a.) or Eq. (24.b.) is (see Section 3.2)

$$t_{v,1/2} = \frac{\ln(2)}{k} = \frac{0.693 \cdot T_S P_{ch} V_{ch}}{T_{ch} P_S \dot{V}_{vex,STP}} \quad (25.a)$$

$$= \frac{\ln(2)}{k_A} = \frac{0.693 \cdot V_{ch}}{\dot{V}_{vex}} \quad (25.b)$$

The ventilation half-time increases with chamber volume and decreases with increasing ventilation rate at the prevailing chamber pressure and temperature.

2.4. CO₂ Accumulation/Washout in an Intermittently Ventilated Constant Temperature Chamber

The P_{CO_2} at the end of the no-ventilation period between $t = 0$ and $t = t_{nv}$ in a no-ventilation – ventilation cycle is given by Eq. (13), with $P_{CO_2}^o$ equal to the chamber P_{CO_2} at the beginning of the cycle:

$$P_{CO_2}(t_{nv}) = \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} t_{nv} + P_{CO_2}^o. \quad (26)$$

The P_{CO_2} at the end of the subsequent ventilation period of duration t_v is given by Eq. (22.a), with P_{CO_2} at the beginning of the ventilation period given by Eq. (26):

$$P_{CO_2}(t_v) = \frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} [1 - e^{-kt_v}] + \left(\frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} t_{nv} + P_{CO_2}^o \right) \cdot e^{-kt_v}. \quad (27)$$

The difference between the end-cycle P_{CO_2} at $t = t_{nv} + t_v$ and the initial P_{CO_2} at $t = 0$ is

$$\Delta_{CO_2} = P_{CO_2}(t_v) - P_{CO_2}^o. \quad (28)$$

Equation (27) is substituted into Eq. (28) to yield

$$\Delta_{CO_2} = \frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} [1 - e^{-kt_v}] + \left(\frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} t_{nv} + P_{CO_2}^o \right) \cdot e^{-kt_v} - P_{CO_2}^o, \quad (29)$$

which is rearranged to obtain

$$\Delta_{CO_2} = \left[\frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} - P_{CO_2}^o \right] \cdot [1 - e^{-kt_v}] + \left(\frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} t_{nv} \right) \cdot e^{-kt_v}. \quad (30)$$

2.4.1. “Large kt_v ” No-Ventilation – Ventilation Steady State

If kt_v is sufficiently large to assume attainment of end-ventilation steady state, then $e^{-kt_v} \rightarrow 0$ and Eq. (30) is reduced to

$$\Delta_{CO_2} = \frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} - P_{CO_2}^o. \quad (31.a)$$

With substitution of Eq. (24.a), this becomes

$$\Delta_{CO_2} = P_{CO_2}(\infty) - P_{CO_2}^o \quad (31.b)$$

Note that Δ_{CO_2} is independent of P_{CO_2} at the end of the no-ventilation period. The latter P_{CO_2} , given by Eq. (26), will be the maximum chamber P_{CO_2} in the no-ventilation – ventilation cycle only with sufficiently high \dot{V}_{vex} during the ventilation period. If ventilation is just sufficient to return chamber P_{CO_2} to its initial value, then $\Delta_{CO_2} = 0$ and successive no-ventilation – ventilation cycles are in a steady state, for which Eq. (31.a) and (31.b) are combined to obtain

$$P_{CO_2}^o = P_{CO_2}(\infty) = \frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} \quad (32)$$

Moreover, if the chamber P_{CO_2} is not returned to its initial value at the end of the first cycle ($\Delta_{CO_2} \neq 0$), a steady state in which the right side of Eq. (32) holds is still achieved in subsequent cycles because they all begin and end with chamber P_{CO_2} equal to $P_{CO_2}(\infty)$. The large kt_v , steady state achieved in either case has minimum chamber P_{CO_2} equal to $P_{CO_2}(\infty)$ which, like Δ_{CO_2} , is independent of the P_{CO_2} at the end of the no-ventilation periods. Thus, t_v and t_{nv} are specified for such steady-state cycles at given chamber pressure after specification of desired P_{CO_2} maxima and minima in the cycles, and with given values of $\dot{V}_{CO_2,met,STP}$ and ventilation gas composition. Eqs. (25.a) and (32) are used to specify t_v in terms of a number of ventilation half-times sufficient to meet the end-ventilation steady-state assumption and the ventilation rate required to maintain the specified $P_{CO_2}^o$. Equation (26.a) or Eq. (26.b) is then used to specify t_{nv} to limit the maximum P_{CO_2} .

It follows from Eq. (22.b) that chamber volume must be replaced many times during ventilation in order to assume attainment of end-ventilation steady state. For example, attainment of $e^{-kt_v} \leq 0.001$ requires $\dot{V}_{ex} t_v \geq 6.9 \cdot V_{ch}$. This requirement is met only with relatively high ventilation rates and times. Specifically, the actual per minute ventilation rate must exceed 70% of the chamber volume for the required ventilation time to be 10 minutes or less. If the actual per minute ventilation rate is only 10% of the chamber volume, the ventilation time must exceed 70 minutes to attain $e^{-kt_v} < 0.001$.

2.4.2. "Small kt_v " No-Ventilation – Ventilation Steady State

If ventilation during the ventilation periods is insufficient to approximate the end-ventilation steady-state assumption, each no-ventilation – ventilation cycle i in a series of n total cycles ends with a $\Delta_{CO_2,i} > 0$. However, the cycle-to-cycle accumulation of chamber CO_2 is progressively attenuated, because the end-ventilation steady state approached at the end of each cycle is independent of chamber P_{CO_2} [see Eqs. (24.a) and (24.b)].

The change in chamber P_{CO_2} during the i^{th} ventilation period is

$$\Delta P_{CO_2}(t_v)_i = q \cdot [P_{CO_2}(\infty) - P_{CO_2}(t_{nv})_i], \quad (33)$$

where $P_{CO_2}(\infty)$ is given by Eq. (24.a) or Eq. (24.b), and q is the fraction of maximum possible washout response, $P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)$, actually achieved during the ventilation period given by

$$q = 1 - 0.5^{(t_v/t_{v,1/2})}, \quad (34)$$

where $t_{v,1/2}$ is the ventilation half-time given by Eq. (25.a) or Eq. (25.b) [see Table 1]. Note that $\Delta P_{CO_2}(t_v)_i < 0$ when the ventilation rate and time are adequate to reduce chamber P_{CO_2} during the ventilation period. Also, q in this context has an absolute lower limit, because it depends on \dot{V}_{vex} [see Eqs. (24.a) and (24.b) and Eqs. (25.a) and (25.b)] and \dot{V}_{vex} is constrained by the constant P_{ch} and $\dot{V}_{CO_2,met,STP}$ assumptions (see Section 3.3).

With $P_{CO_2}^o$ equal to the chamber P_{CO_2} at the beginning of the first no-ventilation period, the chamber P_{CO_2} at the end of the i^{th} no-ventilation period is

$$P_{CO_2}(t_{nv})_i = P_{CO_2}^o + \Delta_{CO_2,i-1} + \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} (t_{nv}), \quad (35)$$

where, for $i = 1$, $\Delta_{CO_2,0} = 0$ and, for subsequent cycles,

$$\Delta_{CO_2,i} = P_{CO_2}(t_{nv})_i + \Delta P_{CO_2}(t_v)_i - P_{CO_2}^o. \quad (36)$$

Equations (33) and (35) are substituted into Eq. (36) to obtain

$$\Delta_{CO_2,i} = \Delta_{CO_2,i-1} + \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} (t_{nv}) + q \cdot [P_{CO_2}(\infty) - P_{CO_2}(t_{nv})_i]. \quad (37)$$

If $P_{CO_2}(t_{nv})_i > P_{CO_2}(\infty)$, the last term in Eq. (37) consists of a constant q times a quantity that becomes increasingly negative in successive cycles. As the number of cycles n approaches infinity under such conditions and the condition $P_{CO_2}(t_{nv})_i < P_{ch}$ holds, this term asymptotically approaches a value equal to the amount of CO_2 added during each no-ventilation period. As a result, $\Delta_{CO_2,i} \rightarrow \Delta_{CO_2,i-1}$, and a different type of no-ventilation – ventilation steady state is approached, a state for which Eq. (37) reduces to

$$q \cdot [P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)] = \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} (t_{nv}). \quad (38)$$

The maximum chamber P_{CO_2} in this “small kt_v ” no-ventilation – ventilation steady state is the P_{CO_2} at the end of each no-ventilation period. This P_{CO_2} is obtained for the i^{th} period by rearranging Eq. (38) to obtain

$$P_{CO_2}(t_{nv})_i = P_{CO_2}(\infty) + \left\{ \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} \left(\frac{t_{nv}}{q} \right) \right\}. \quad (39)$$

The minimum chamber P_{CO_2} in this steady state is that at the end of each ventilation period and that given by

$$P_{CO_2}(t_v)_i = P_{CO_2}^o + \Delta_{CO_2,i}, \quad (40)$$

an expression that, after Eqs. (33) and (36) are substituted, becomes

$$P_{CO_2}(t_v)_i = P_{CO_2}(t_{nv})_i - q[P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)]. \quad (41)$$

An alternative expression for the maximum chamber P_{CO_2} in this steady state is obtained by noting that

$$q \cdot [P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)] = P_{CO_2}(t_{nv})_i - P_{CO_2}(t_v)_i. \quad (42)$$

Substituting Eq. (22.a) into Eq. (42) then yields

$$q \cdot [P_{CO_2}(t_{nv})_i - P_{CO_2}(\infty)] = P_{CO_2}(t_{nv})_i - \left[\frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} [1 - e^{-kt_v}] + P_{CO_2}(t_{nv})_i \cdot e^{-kt_v} \right], \quad (43)$$

which is substituted into Eq. (38) to obtain

$$\frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}}{T_S V_{ch}} (t_{nv}) = P_{CO_2}(t_{nv})_i - \left[\frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} [1 - e^{-kt_v}] + P_{CO_2}(t_{nv})_i \cdot e^{-kt_v} \right]. \quad (44)$$

Finally, Eq. (44) is solved for $P_{CO_2}(t_{nv})_i$ to obtain:

$$P_{CO_2}(t_{nv})_i = \frac{T_{ch} P_S \dot{V}_{CO_2,met,STP}(t_{nv})}{T_S V_{ch} [1 - e^{-kt_v}]} + \left[\frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} \right], \quad (45.a)$$

or, with the ventilation rate expressed as the volume flux at actual chamber temperature and pressure,

$$P_{CO_2}(t_{nv})_i = \frac{T_{ch} P_S}{T_S} \left[\frac{\dot{V}_{CO_2,met,STP}(t_{nv})}{V_{ch} [1 - e^{-k_A t_v}]} + \frac{\{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex}} \right]. \quad (45.b)$$

Alternatively, Eq. (44) is solved for t_{nv} to obtain the no-ventilation time required to sustain a given maximum chamber P_{CO_2} with given ventilation time and $\dot{V}_{CO_2,met,STP}$:

$$t_{nv} = \frac{T_S V_{ch} [1 - e^{-kt_v}]}{T_{ch} P_S \dot{V}_{CO_2,met,STP}} \left[P_{CO_2}(t_{nv})_i - \frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}} \right], \quad (46.a)$$

or, with the ventilation rate expressed as the volume flux at actual chamber temperature and pressure,

$$t_{nv} = \frac{V_{ch} [1 - e^{-k_A t_v}]}{\dot{V}_{CO_2,met,STP}} \left[\frac{T_S P_{CO_2}(t_{nv})_i}{T_{ch} P_S} - \frac{\{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex}} \right]. \quad (46.b)$$

2.5. CO₂ Accumulation in a Continuously CO₂-Scrubbed Constant Temperature Chamber

The analog of Eq. (14) to consider CO₂ content in a constant-pressure chamber with an internal scrubber that continuously removes CO₂ is

$$\left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_A} = \frac{T_{ch} P_S}{T_S V_{ch}} \left(\dot{V}_{CO_2,met,STP} - \{ \dot{V}_{CO_2,in,STP} - \dot{V}_{CO_2,out,STP} \} \right), \quad (47)$$

where terms in the curly brackets denote the action of the scrubber — i.e., the change in partial volume of CO₂ in the overall ventilation of the scrubber, $\dot{V}_{vent,STP}$. Note that the sense of the “in” and “out” subscripts is different from that used when chamber ventilation is considered. Here, $\dot{V}_{CO_2,in,STP}$ is the standard CO₂ partial volume flow into the scrubber, and $\dot{V}_{CO_2,out,STP}$ is the standard CO₂ partial volume flow in the scrubber exhaust. The analog of Eq. (15) for these conditions is consequently

$$\dot{V}_{CO_2,in,STP} = \frac{P_{CO_2}}{P_{ch}} \dot{V}_{vin,STP}, \quad (48)$$

where $\dot{V}_{vin,STP}$ is the standard scrubber input ventilation rate. We then assume that the standard CO_2 partial volume in the scrubber exhaust is a constant fraction f ($0 \leq f < 1$) of the standard CO_2 partial volume entering the scrubber:

$$\dot{V}_{CO_2,out,STP} = f \cdot \dot{V}_{CO_2,in,STP} = \frac{P_{CO_2}}{P_{ch}} \dot{V}_{vin,STP} \cdot f. \quad (49)$$

Rearrangement after Eqs. (48) and (49) are substituted into Eq. (47) yields:

$$\left(\frac{\partial P_{CO_2}}{\partial t} \right)_{T_{ch}} + \left(\frac{T_{ch} P_S F \cdot \dot{V}_{vin,STP}}{T_S P_{ch} V_{ch}} \right) P_{CO_2} = \frac{T_{ch} P_S}{T_S V_{ch}} \dot{V}_{CO_2,met,STP}, \quad (50)$$

where $F = (1 - f)$ is the scrubber efficiency factor. Equation (50) is identical to Eq. (17), with $\dot{V}_{CO_2,in,STP} = 0$ and $\dot{V}_{vex,STP}$ replaced by $F \cdot \dot{V}_{vin,STP}$. With constant $F \cdot \dot{V}_{vin,STP}$ and P_{ch} , the solution of this equation is consequently obtained as before:

$$P_{CO_2}(t) = \frac{P_{ch} \dot{V}_{CO_2,met,STP}}{F \cdot \dot{V}_{vin,STP}} [1 - e^{-kt}] + P_{CO_2}^o e^{-kt}, \quad \text{with } k = \frac{T_{ch} P_S F \cdot \dot{V}_{vin,STP}}{T_S P_{ch} V_{ch}}; \quad (51.a)$$

and

$$P_{CO_2}(t) = \frac{P_S T_{ch} \dot{V}_{CO_2,met,STP}}{T_S \cdot F \cdot \dot{V}_{vin}} [1 - e^{-k_A t}] + P_{CO_2}^o e^{-k_A t}, \quad \text{with } k_A = \frac{F \cdot \dot{V}_{vin}}{V_{ch}}; \quad (51.b)$$

where Eq. (51.a) is for use when the scrubber ventilation is expressed at STP, and Eq. (51.b) is for use when the scrubber ventilation is expressed at actual chamber temperature and pressure.

Note that maintenance of the constant P_{ch} required by this equation requires a constant O_2 bleed into the chamber at standard rate equal to $\dot{V}_{O_2,met,STP}$, while with constant $F \cdot \dot{V}_{vin,STP}$, $\dot{V}_{CO_2,scrub,STP}$ varies with the chamber CO_2 mole fraction.

Similar to the situation in a continuously ventilated chamber, the P_{CO_2} in a continuously scrubbed chamber approaches a steady-state value given by the respective solutions of Eq. (51.a) and Eq. (51.b) at $t = \infty$:

$$P_{CO_2}(\infty) = \frac{P_{ch} \dot{V}_{CO_2,met,STP}}{F \cdot \dot{V}_{vin,STP}} \quad (52.a)$$

$$= \frac{P_S T_{ch} \dot{V}_{CO_2,met,STP}}{T_S \cdot F \cdot \dot{V}_{vin}}, \quad (52.b)$$

where $F \cdot \dot{V}_{vin,STP} \geq \dot{V}_{CO_2,met,STP}$. The minimum scrubber ventilation \times efficiency factor product stipulated by Eqs. (52.a) and (52.b) satisfies both physiological and constant P_{ch} requirements at the required chamber input bleed of 100% O₂ (see Section 3.3).

The half-time $t_{v,1/2}$ for approach to the steady state given by Eqs. (52.a) and (52.b) is (see Section 3.2):

$$t_{v,1/2} = \frac{\ln(2)}{k} = \frac{0.693 \cdot T_S P_{ch} V_{ch}}{T_{ch} P_S F \cdot \dot{V}_{vin,STP}} \quad (53.a)$$

$$= \frac{\ln(2)}{k_A} = \frac{0.693 \cdot V_{ch}}{F \cdot \dot{V}_{vin}} \quad (53.b)$$

3. SUPPLEMENTAL MATERIAL

3.1. Derivation of Equation (19), Solution of the Differential Equation $y' + ky = c$

The present theory of CO₂ accumulation in ventilated or CO₂-scrubbed systems is based on the solution of a simple first-order differential equation $y' + ky = c$, where y is a function of t ($y \equiv y(t)$), $y' \equiv dy/dt$, and k and c are constants. This solution is a standard problem in first courses in differential equations,² but is presented here for completeness.

We start by multiplying both sides of the equation by the integrating factor e^{kt} to obtain

$$e^{kt} (y' + ky) = ce^{kt}, \quad (3.1)$$

which simplifies to

$$(ye^{kt})' = ce^{kt}. \quad (3.2)$$

Equation (3.2) is integrated to obtain

$$ye^{kt} = \int ce^{kx} dx + C, \quad (3.3)$$

where C is an integration constant, the value of which is governed by the initial conditions for the problem. Eq. (3.3) is rearranged, and the integral is evaluated to obtain

$$y = ce^{-kt} \int e^{kx} dx + Ce^{-kt} = \frac{ce^{-kt} (e^{kx} |_0^t)}{k} + Ce^{-kt} \quad (3.4)$$

and

$$y = \frac{ce^{-kt} (e^{kt} - 1)}{k} + Ce^{-kt} \quad (3.5)$$

Eq. (3.5) is rearranged to obtain our final result:

$$y = \frac{c(1 - e^{-kt})}{k} + Ce^{-kt} \quad (3.6)$$

3.2. Ventilation Half-Time Derivation

Continuous chamber or CO₂ scrubber ventilation started at $t = 0$ with initial chamber P_{CO_2} equal to $P_{CO_2}(t_o)$ causes a maximum chamber P_{CO_2} change equal to $P_{CO_2}(t_\infty) - P_{CO_2}(t_o)$ at $t = t_\infty$. The fraction q of this total response attained in period t_q beginning at t_o is

$$q = \frac{P_{CO_2}(t_o + t_q) - P_{CO_2}(t_o)}{P_{CO_2}(t_\infty) - P_{CO_2}(t_o)} = \frac{Z[1 - e^{-k(t_o + t_q)}] + P_{CO_2}^o e^{-k(t_o + t_q)} - Z[1 - e^{-kt_o}] - P_{CO_2}^o e^{-kt_o}}{Z[1 - e^{-kt_\infty}] + P_{CO_2}^o e^{-kt_\infty} - Z[1 - e^{-kt_o}] - P_{CO_2}^o e^{-kt_o}}, \quad (3.7)$$

where $Z \equiv \frac{P_{ch} \{ \dot{V}_{CO_2,met,STP} + \dot{V}_{CO_2,in,STP} \}}{\dot{V}_{vex,STP}}$ for the ventilated chamber, and

$$Z \equiv \frac{P_{ch} \dot{V}_{CO_2,met,STP}}{F \cdot \dot{V}_{vin,STP}} \quad \text{for the scrubbed chamber.}$$

In general, $0 \leq q \leq 1$, but q has lower bounds > 0 with the present definitions of Z .

Equation (3.7) is solved for t_q to obtain the time required to attain fraction q of the total remaining response at $t = t_o$:

$$q = \frac{Z[1 - e^{-kt_o} e^{-kt_q}] + P_{CO_2}^o e^{-kt_o} e^{-kt_q} - Z[1 - e^{-kt_o}] - P_{CO_2}^o e^{-kt_o}}{Z - Z[1 - e^{-kt_o}] - P_{CO_2}^o e^{-kt_o}}$$

$$q = \frac{-Ze^{-kt_o}e^{-kt_q} + P_{CO_2}^o e^{-kt_o}e^{-kt_q} + Ze^{-kt_o} - P_{CO_2}^o e^{-kt_o}}{Ze^{-kt_o} - P_{CO_2}^o e^{-kt_o}} = \frac{(P_{CO_2}^o - Z)e^{-kt_o}e^{-kt_q} + (Z - P_{CO_2}^o)e^{-kt_o}}{(Z - P_{CO_2}^o)e^{-kt_o}}$$

$$q = \frac{(Z - P_{CO_2}^o)e^{-kt_o} - (Z - P_{CO_2}^o)e^{-kt_o}e^{-kt_q}}{(Z - P_{CO_2}^o)e^{-kt_o}} = 1 - e^{-kt_q}$$

$$1 - q = e^{-kt_q}$$

$$-kt_q = \ln(1 - q)$$

$$t_q = \frac{-\ln(1 - q)}{k} = \frac{-\ln Q}{k}, \quad (3.8)$$

where $Q = 1 - q$ is the fraction of the $t_o \rightarrow t_\infty$ response remaining at the end of the t_q period, $t_o + t_q$. For the half-time, the time required to attain one-half of the remaining response at $t = t_o$, $q = 1/2$ and

$$t_{1/2} = \frac{-\ln(0.5)}{k} = \frac{\ln 2}{k}. \quad (3.9)$$

Values of q , Q , and $-\ln(Q)$ for various multiples of $t_{1/2}$ are given in Table 1.

Table 1.
Values of q , Q , and $-\ln(Q)$ for various multiples of $t_{1/2}$

# half-times ($t_v/t_{v,1/2}$)	response fraction at t_v q	remaining response fraction at t_v Q	Q , %	$-\ln(Q)$
0	0.000	1.000	100.000	0.000
1	0.500	0.500	50.000	0.693
2	0.750	0.250	25.000	1.386
3	0.875	0.125	12.500	2.079
4	0.938	0.063	6.250	2.773
5	0.969	0.031	3.125	3.466
6	0.984	0.016	1.563	4.159
7	0.992	0.008	0.781	4.852
8	0.996	0.004	0.391	5.545
9	0.998	0.002	0.195	6.238
10	0.999	0.001	0.098	6.931

3.3. Consequences of Constant P_{ch} and $\dot{V}_{CO_2,met}$ Assumptions

Present analytic solutions to the equations for $P_{CO_2}(t)$ in ventilated and CO_2 -scrubbed chambers [Eqs. (22.a) and (22.b) and Eqs. (51.a) and (51.b), respectively] and all dependent results require constant chamber pressure, P_{ch} , and a constant rate of metabolic CO_2 production by the chamber occupants, $\dot{V}_{CO_2,met}$. We here consider minimum chamber input, chamber exhaust, and CO_2 scrubber ventilation rates required to meet these assumptions, regardless of whether the chamber is considered "ventilated," because these minimum rates preclude consideration of cases that otherwise yield undefined or nonsensical results [e.g., $P_{CO_2}(t) > P_{ch}$ in Eqs. (22.a) and (22.b) and in Eqs. (51.a) and (51.b)].

Equation (1) is solved for P_{ch} , and then differentiated with respect to time at constant chamber temperature, to obtain

$$\left(\frac{\partial P_{ch}}{\partial t}\right)_{T_{ch}} = \frac{RT_{ch}}{V_{ch}} \left(\frac{\partial n_T}{\partial t}\right)_{T_{ch}} \quad (3.10)$$

With $\left(\frac{\partial P_{ch}}{\partial t}\right)_{T_{ch}} = 0$ and $\left(\frac{\partial n_T}{\partial t}\right)_{T_{ch}, P_{ch}} = \sum \left(\frac{\partial n_g}{\partial t}\right)_{T_{ch}, P_{ch}} \equiv \sum \dot{n}_g \equiv \dot{n}_T$, where each summation is over the all gas species g in the mix comprising n_T , mass balance requires that

$$\frac{RT_{ch}}{V_{ch}} \left\{ \dot{n}_{T,in} + \sum \dot{n}_{g,met} + \sum \dot{n}_{g,scrub} \right\} = \frac{RT_{ch}}{V_{ch}} \dot{n}_{T,out} \quad (3.11)$$

where $\sum \dot{n}_{g,met}$ is the sum of changes in the chamber gas content due to the metabolic activity of

the chamber occupants, and $\sum \dot{n}_{g,scrub}$ is the sum of changes in the chamber gas content due to the

action of a CO_2 scrubber. Equation (1) for the molar amounts of chamber input and exhaust gases is differentiated at constant temperature and pressure to obtain, respectively:

$$\dot{n}_{T,in} = \frac{P_{in}}{RT_{ch}} \left(\frac{\partial V_{T,in}}{\partial t}\right)_{T_{ch}, P_{in}} \equiv \frac{P_{in}}{RT_{ch}} \dot{V}_{vin} \quad (3.12)$$

and

$$\dot{n}_{T,out} = \frac{P_{ex}}{RT_{ch}} \left(\frac{\partial V_{T,ex}}{\partial t}\right)_{T_A, P_{ex}} \equiv \frac{P_{ex}}{RT_{ch}} \dot{V}_{vex} \quad (3.13)$$

where P_{in} and P_{ex} are the respective pressures of the input and exhaust ventilation flows. If the input and exhaust ventilation rates are expressed at chamber pressure, then $P_{in} = P_{ex} = P_{ch}$ and Eqs. (3.12) and (3.13) are substituted into Eq.(3.11) to obtain

$$\dot{V}_{vin}P_{ch} + RT_{ch} \left\{ \sum \dot{n}_{g,met} + \sum \dot{n}_{g,scrub} \right\} = \dot{V}_{vex}P_{ch}. \quad (3.14)$$

If the chamber humidity is assumed to be constant, $\dot{n}_{g,met} = 0$ for all gases except O_2 and CO_2 . For O_2 , we have

$$\dot{n}_{O_2,met} = -\frac{P_{ch}\dot{V}_{O_2,met}}{RT_{ch}}, \quad (3.15)$$

and for CO_2 ,

$$\dot{n}_{CO_2,met} = \frac{P_{ch}\dot{V}_{O_2,met}RQ}{RT_{ch}}, \quad (3.16)$$

where we have invoked the definition of the respiratory quotient, $RQ = \dot{V}_{CO_2,met} / \dot{V}_{O_2,met}$. Equations (3.15) and (3.16) are substituted into Eq. (3.14) to obtain

$$\dot{V}_{vin} + \dot{V}_{O_2,met}(RQ - 1) + \frac{RT_{ch}}{P_{ch}} \sum \dot{n}_{g,scrub} = \dot{V}_{vex}. \quad (3.17)$$

Input and exhaust ventilation rates, as well as any CO_2 scrubbing rate, must satisfy Eq. (3.17) to maintain constant P_{ch} .

No CO_2 Scrubbing

With no scrubbing, the absolute minimum ventilation rates required to keep P_{ch} constant are obtained by rearranging Eq. (3.17) with $\sum \dot{n}_{g,scrub} = 0$:

$$\dot{V}_{vin} = \dot{V}_{O_2,met}(1 - RQ), \text{ with } RQ \leq 1 \text{ and } \dot{V}_{vex} = 0, \quad (3.18.a)$$

or

$$\dot{V}_{vex} = \dot{V}_{O_2,met}(RQ - 1), \text{ with } RQ \geq 1 \text{ and } \dot{V}_{vin} = 0. \quad (3.18.b)$$

Because $\dot{V}_{vin} < \dot{V}_{O_2,met}$ in either case, the absolute minimum ventilations cannot indefinitely sustain any specified $\dot{V}_{O_2,met}$, and the constant $\dot{V}_{CO_2,met}$ assumption must ultimately be violated. This

violation must be avoided to consider the solutions of Eqs. (22.a) and (22.b) and Eqs. (51.a) and (51.b) at $t = \infty$ [see Eqs. (24.a) and (24.b) and Eqs. (52.a) and (52.b)]. Thus, indefinite maintenance of any specified $\dot{V}_{O_2,met}$ requires a minimum input ventilation of 100% O_2 at a rate equal to $\dot{V}_{O_2,met}$. This minimum input, however, exceeds the minimum \dot{V}_{vin} in either Eq. (3.18.a) or Eq. (3.18.b), so we then have the requirement that $\dot{V}_{vex} > 0$ to keep P_{ch} constant. Under these circumstances, Eq. (3.17) becomes

$$\dot{V}_{vex} = \dot{V}_{O_2,met} + \dot{V}_{O_2,met}(RQ - 1), \quad (3.19)$$

which reduces to

$$\dot{V}_{vex} = \dot{V}_{O_2,met} \cdot RQ = \dot{V}_{CO_2,met}, \text{ with } \dot{V}_{vin}(100\% O_2) = \dot{V}_{O_2,met}. \quad (3.20)$$

CO₂ Scrubbing

The absolute minimum scrubber ventilation rate required to keep P_{ch} constant in a scrubbed chamber is obtained from Eq. (3.17) with $\dot{V}_{vex} = 0$ and $\dot{V}_{vin}(100\% O_2) = \dot{V}_{O_2,met}$ to indefinitely sustain any assumed $\dot{V}_{O_2,met}$. Under these conditions, Eq. (3.17) reduces to

$$\dot{V}_{O_2,met} RQ = F \cdot \dot{V}_{vin,scr} \left(\frac{P_{CO_2}}{P_{ch}} \right), \quad (3.21)$$

where the definition of $\frac{RT_{ch}}{P_{ch}} \sum \dot{n}_{g,scrub}$ from Eqs. (48) and (49) has been substituted. Equation (3.21) is solved for $\dot{V}_{vin,scr}$ to obtain

$$\dot{V}_{vin,scr} = \frac{P_{ch} \dot{V}_{O_2,met} RQ}{F \cdot P_{CO_2}} = \frac{P_{ch} \dot{V}_{CO_2,met}}{F \cdot P_{CO_2}}, \text{ with } \dot{V}_{vin}(100\% O_2) = \dot{V}_{O_2,met}. \quad (3.22)$$

The absolute minimum scrubber ventilation rate, $\dot{V}_{vin,scr} = \dot{V}_{CO_2,met} / F$, occurs in the worst case, where $P_{CO_2} = P_{ch}$. Greater scrubber ventilation rates drive the chamber P_{CO_2} downward toward the steady-state value given by Eq. (52.a) or Eq. (52.b). Note that chamber input ventilation rates greater than $\dot{V}_{O_2,met}$ require that $\dot{V}_{vex} > 0$. The chamber is then both scrubbed and ventilated, and Eqs. (51.a) and (51.b) no longer hold. Consideration of these combined conditions is beyond our present scope.

4. SUMMARY

Analytic expressions have been derived to compute the value of the atmospheric P_{CO_2} in well-mixed, constant-volume manned diving systems during various operational conditions at a constant temperature T_{ch} .

Equation (13) gives the system P_{CO_2} at any time t in a no-ventilation period between $t = 0$ and $t = t_{nv}$ with initial $P_{CO_2} = P_{CO_2}^o$.

If the diving system remains at constant pressure P_{ch} , Eq. (22.a) or Eq. (22.b) gives the system P_{CO_2} at any time t in a continuous ventilation period between $t = 0$ and $t = t_v$ with initial $P_{CO_2} = P_{CO_2}^o$.

During such a ventilation period, the system asymptotically approaches a steady-state P_{CO_2} given by Eq. (24.a) or Eq. (24.b), with half-time given by Eq. (25.a) or Eq. (25.b).

Intermittent ventilation with constant no-ventilation and ventilation times at constant system pressure causes system P_{CO_2} to oscillate between maximum and minimum values in successive no-ventilation – ventilation cycles. Over time, these maxima and minima asymptotically approach steady-state values, with maximum system P_{CO_2} given by Eq. (45.a) or Eq. (45.b) and minimum system P_{CO_2} given by Eq. (41). The no-ventilation time required to sustain a given maximum system P_{CO_2} at given ventilation time and $\dot{V}_{CO_2,met,STP}$ is given by Eq. (46.a) or Eq. (46.b).

If the diving system remains at a constant pressure P_{ch} while CO_2 is continuously scrubbed from the system atmosphere, Eq. (51.a) or Eq. (51.b) gives the system P_{CO_2} at any time t in a continuous ventilation period between $t = 0$ and $t = t_v$ with an initial $P_{CO_2} = P_{CO_2}^o$. During such a scrubbing period, the system P_{CO_2} asymptotically approaches a steady state value given by Eq. (52.a) or Eq. (52.b), with half-time given by Eq. (53.a) or Eq. (53.b).

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5. REFERENCES

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